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SENSITIVITY OF FINAL FIELD POSITION TO THE PUNT INITIAL CONDITIONS IN AMERICAN FOOTBALL

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ABSTRACT

The starting field position is often a deciding factor in an American football game. In the case of a defensive stop, a kick, known as a punt, is used to give the receiving team a field position that is more advantageous to the kicking team when possession changes. The goal of the punter is to kick the ball along a desired flight path, where a delicate balance between the distance traveled before impact, hang time in the air, and the distance traveled after bouncing is favorable for the kicking team. However, the punter has only imprecise control over the initial conditions, such as the angular velocity, linear velocity, and orientation of the football. Due to the highly nonlinear behavior of the football, from aerodynamic and impact forces, even small changes in initial conditions can produce large changes in the final position of the football, but there may be regions of initial conditions with relatively consistent results. If punters could target such large contiguous regions of initial conditions with desirable football paths, they could improve their chances of successful kicks.

For nonlinear systems, basins of attraction diagrams are often used to graphically display the initial conditions that lead to different final attractors. In this case, the regions of initial conditions that lead to a desirable final field position can be grouped and shown graphically. A numerical simulation program was developed including models for aerodynamic flight and bouncing of the irregularly shaped football. The flight model used fourth order Runge–Kutta integration of the equations of motion of the football, including gravitational and aerodynamic forces and moments with empirical lift, drag, and yaw coefficients in three dimensions. The bounce model was based on an empirical two-dimensional coefficient of restitution model that was published in the literature. The behavior of a football in flight and during bouncing was simulated for a range of initial angular velocities and launch angles, and the characteristics of the flight paths were analyzed. The characteristics of some regions of initial conditions were relatively sensitive to small changes, while other regions were relatively uniform. This shows that this approach, with a quantitatively accurate bounce model, could be practically applied to develop a guide for punters to optimize their kicks. With such a guide and sufficient practice, punters could select and target the larger regions of initial conditions that produced desirable behavior, which would improve their chances of successful punts.

NOMENCLATURE

$\hat{\boldsymbol{\imath}}_B, \hat{\boldsymbol{\jmath}}_B, \hat{\boldsymbol{k}}_B$ Unit vectors describing the B-frame.
$\hat{\boldsymbol{\imath}}_O, \hat{\boldsymbol{\jmath}}_O, \hat{\boldsymbol{k}}_O$ Unit vectors describing the O-frame.
t Time from the start of the simulation.
x_O, y_O, z_O Position of the ball in the O-frame.
v_{xB}, v_{yB}, v_{zB} Velocity of the ball in the B-frame.
$\omega_x, \omega_y, \omega_z$ Angular velocity of the ball in the B-frame.
$e_0,\ e_1,\ e_2,\ e_3$ Components of unit quaternion describing the orientation of B-frame relative to the O-frame.

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INTRODUCTION

The starting field position is often a deciding factor in the outcome of an American football game. There are multiple types of kicks in football, including field goals, punts, and kickoffs. The goal of the kicker is to kick the ball along a desired flight path favorable for their team, balancing between the distance traveled before impact, hang time in the air, and the distance traveled after bouncing. Depending on the type of kick, the distance may be more important than the hang time, or vice versa. For example, Brancazio determined that for kickoffs, typical launch angles are around 45° to maximize distance, while for punts, typical launch angles are more variable and range around 55° to 60° to achieve greater hang time [1]. By making a kick with the optimal distance and hang time, a kicker can significantly help their team.

However, the kicker has only imprecise control over the initial conditions of the football, such as its angular velocity, linear velocity, and orientation. Due to the highly nonlinear behavior of the football, from aerodynamic and impact forces, even small changes in initial conditions can produce large changes in the final position of the football, but there may be regions of initial conditions with relatively consistent results. If kickers could target such large contiguous regions of initial conditions with desirable paths, they could improve their chances of successful kicks. To identify these regions, it is necessary to consider both the flight and bouncing of the football.

The motion of the football through the air is quite complex and must account for aerodynamic forces and moments, which can be on the same order of magnitude as the weight of the football [1]. The aerodynamic forces depend not only on the velocity but also on orientation of the football due to its nonspherical shape. The orientation of the football changes throughout the flight, often demonstrating precession, and aerodynamic drag can vary by as much as an order of magnitude between different orientations [2]. Measurements of aerodynamic coefficients of the American football and the rugby ball have been performed by several researchers [3, 4, 5]. Several researchers have also developed models of the football and rugby ball in flight, with the most general model developed by Lee et al. [5, 6, 7]. Since the model developed by Lee et al. [7] incorporates the measurements of [3, 5] and allows for general motion, it was used for this study.

The ellipsoidal shape of the football also complicates its bouncing behavior. Variations of the mechanical properties of the football and turf under different conditions can also affect the bouncing of the football [8] but were not considered in this study. Oblique collisions of spherical balls against flat surfaces have been fairly extensively studied, see e.g. [9, 10, 11, 12, 13, 14], but collisions of ellipsoidal balls are less understood. Cross performed the only known study on the physics of football bouncing, taking measurements with high speed photography of a football bouncing within a single plane of motion [15]. These measurements provide enough information to develop a very rough empirical model for bouncing.

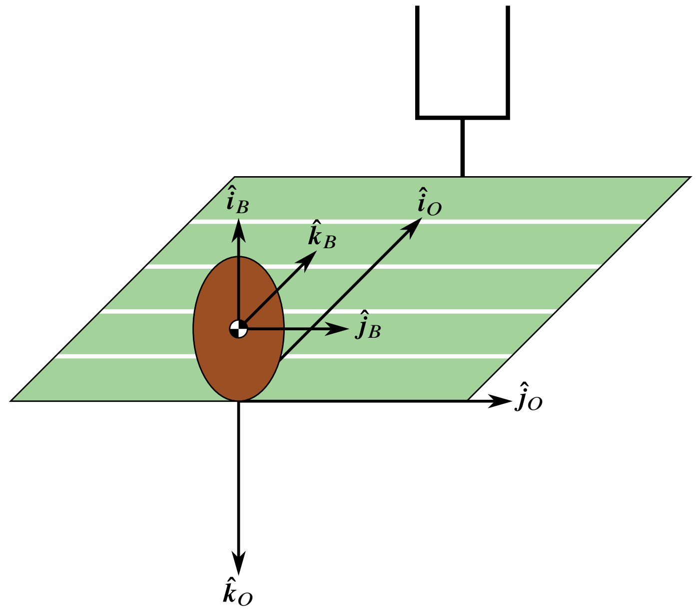

FIGURE 1: SCHEMATIC OF COORDINATE SYSTEMS

While the flight and bouncing of the football have been studied separately in the literature, they have not previously been studied together in a combined model to determine the effect of initial conditions. In particular, the sensitivity of the football's behavior to initial conditions was studied for a range of initial conditions to determine whether it would be feasible for a kicker to kick the football a consistent distance even after bouncing.

MATH MODEL

A combined numerical model accounting for flight and bouncing of the football was developed based on the work of Lee et al. and R. Cross [7, 15]. The model was integrated in time to determine the motion of the football for the initial conditions.

Flight

The flight of the football was simulated using a model developed by Lee et al. which includes the effects of gravity and aerodynamic loads [7]. A body-fixed coordinate system $B = \left\{\hat{\imath}_B, \hat{\jmath}_B, \hat{k}_B\right\}$ and global coordinate system $O = \left\{\hat{\imath}_O, \hat{\jmath}_O, \hat{k}_O\right\}$ were used in the model, as illustrated in Fig. 1. The orientation of the football relative to the global coordinate system was represented with a unit quaternion composed of e_0, e_1, e_2, e_3. The state of the football was described by a vector $\boldsymbol{x}$, where

$$\boldsymbol{x} = \begin{bmatrix} x_O & y_O & z_O & v_{xB} & v_{yB} & v_{zB} & \omega_x & \omega_y & \omega_z & e_0 & e_1 & e_2 & e_3 \end{bmatrix}^\top \quad (1)$$

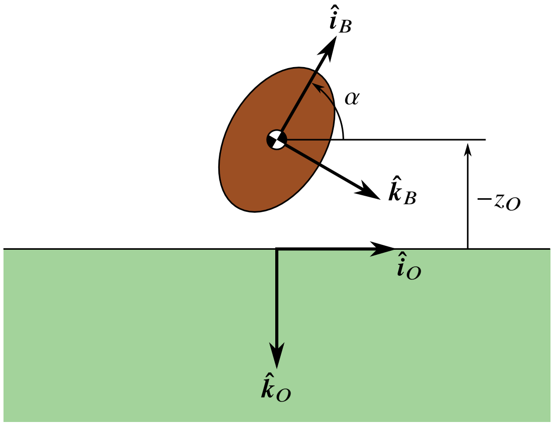

FIGURE 2: SCHEMATIC OF COLLISION DETECTION METHOD

During flight, the motion of the ball was described by the autonomous first order vector differential equation

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) \tag{2}$$

where the components of $\boldsymbol{f}$ are defined in [7]. The solution to the differential equation was approximated by fourth order Runge–Kutta integration with a time step of 5 ms.

Collision Detection

Collisions of the football with the ground were detected by approximating the cross section of the football as an oval. It can be shown that the ball intersects the ground if and only if

$$z_O \geq 0 \quad \text{or} \quad (r_{\text{major}} \sin\alpha)^2 + (r_{\text{minor}} \cos\alpha)^2 - z_O^2 \geq 0 \tag{3}$$

where the orientation α is defined as in Fig. 2, $r_{\text{major}} \approx 0.143\,\text{m}$ is the major radius of the cross section of the football, and $r_{\text{minor}} \approx 0.090\,\text{m}$ is the minor radius of the cross section of the football [15]. When a collision was detected during the integration of Eqn. (2), the simulation program backtracked one time step and then integrated with a smaller time step of 10 µs to more precisely determine the time of the collision.

Bouncing

An empirical coefficient of restitution model interpolating/extrapolating the limited experimental data from [15] was chosen to simulate the bouncing of the football. The model relates the incoming orientation, velocity, and angular velocity of the football to the outgoing orientation, velocity, and angular velocity after the bounce. The model is limited to motion and orientation in the x_O–z_O plane, shown in Fig. 2, so all numerical studies were limited to motion in that plane.

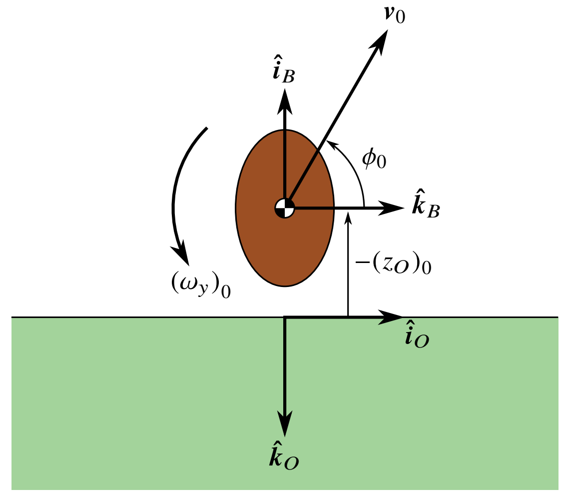

FIGURE 3: SCHEMATIC OF INITIAL CONDITIONS

NUMERICAL STUDIES

The initial conditions for the numerical studies, illustrated in Fig. 3, were chosen such that the motion of the football stayed in the x_O–z_O plane. The values of the initial conditions were based on realistic values from [7]. The initial conditions chosen were

$$\boldsymbol{x}\Big|_{t=0} = \begin{bmatrix} 0 & 0 & (z_O)_0 & (v_{xB})_0 & 0 & (v_{zB})_0 & 0 & (\omega_y)_0 & 0 & \frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \end{bmatrix}^{\top} \tag{4}$$

where

$$(v_{xB})_0 = v_0 \sin\phi_0 \tag{5}$$
$$(v_{zB})_0 = v_0 \cos\phi_0 \tag{6}$$
$$(z_O)_0 = -0.3\,\text{m} \tag{7}$$
$$v_0 = 35.6\,\text{m/s} \tag{8}$$

This describes a vertically oriented football at a short distance off the ground launched at an upward angle with backspin, as shown in Fig. 3. The initial conditions ϕ_0 and $(\omega_y)_0$ were varied for this study.

Trajectories

Trajectories of the football were simulated for a variety of initial conditions. One such trajectory is shown in Fig. 4, and the first bounce in shown more closely in Fig. 5. The initial conditions used for this trajectory are provided in Eqn. (4) with $\phi_0 = 46°$ and $(\omega_y)_0 = 20.22\,\text{rad/s}$.

Most of the distance is covered before first bounce, but the football also travels a fairly significant distance afterward. The effect of drag is clearly visible in Fig. 4, where the trajectory from

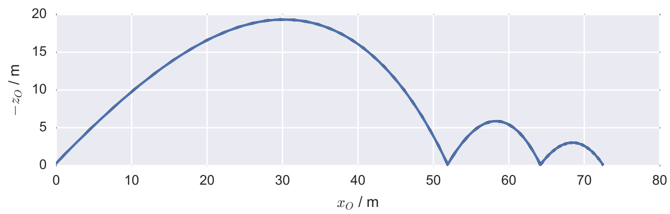

FIGURE 4: SAMPLE TRAJECTORY

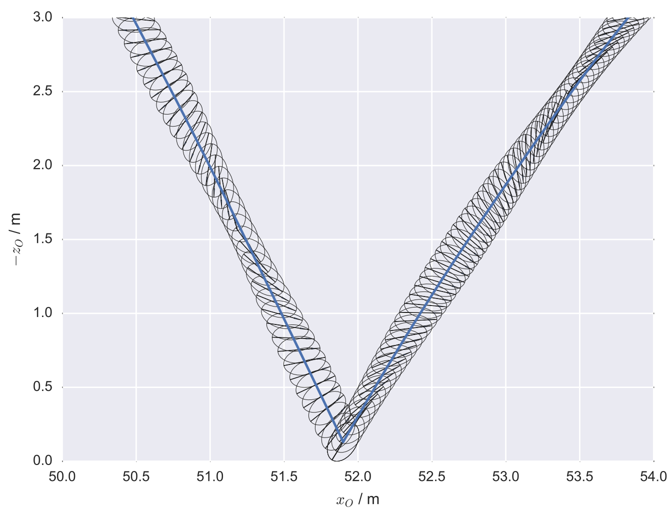

FIGURE 5: FIRST BOUNCE OF FIG. 4; LINE FOLLOWS CG OF FOOTBALL AND ARROWS ARE $\hat{\boldsymbol{\imath}}_B$.

the initial state to the first bounce is clearly asymmetric. Also note that the effect of drag appears to be the most severe when the velocity is highest.

The first bounce shown in Fig. 5 shows that the velocity and angular velocity can change significantly during the bounce. The incoming angular velocity was in the positive $\hat{\boldsymbol{\jmath}}_O$ direction, while the outgoing angular velocity was in the entirely opposite direction. Additionally, the magnitude of the linear velocity noticeably decreased. While the direction of the linear velocity in this case was approximately a reflection, in many cases the direction is significantly changed, and in some cases, even backward.

Also interesting are the kinetic energy $\mathcal{T}$ and potential energy $\mathcal{U}$ of the football, which are shown in Fig. 6. Energy was transformed between kinetic and potential during flight, but total energy was lost due to aerodynamic and collision effects. The aerodynamic losses were most severe when the velocity was highest, such as from 0 s to 1 s; this matches the trajectory in Fig. 4. The energy lost in collisions was also greatest when the impact velocity was the greatest, which matches physical intuition. The

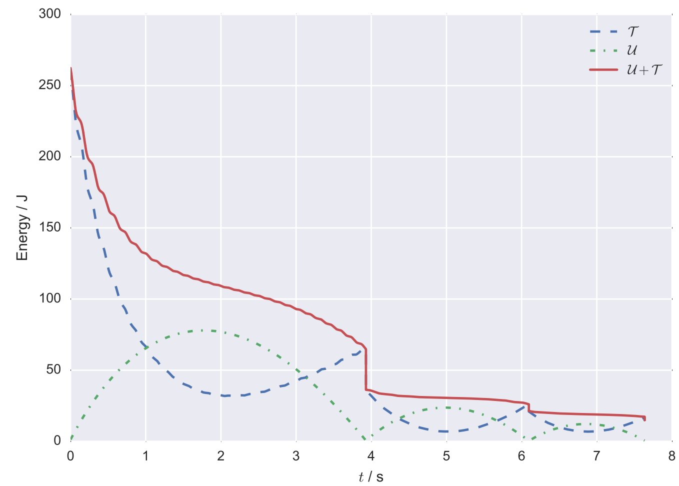

FIGURE 6: KINETIC, POTENTIAL, AND TOTAL ENERGY OF THE FOOTBALL FOR THE TRAJECTORY IN FIG. 4.

simulation was terminated when the total energy of the football was less than 15 J.

Distance Traveled

For nonlinear systems, basins of attraction diagrams are often used to graphically display the initial conditions that lead to different final attractors. In this case, the regions of initial conditions that lead to a desirable final field position can be grouped and shown graphically.

Trajectories of the football were simulated for 40 000 combinations of ϕ_0 and $(\omega_y)_0$. The distance traveled in the $\hat{\boldsymbol{\imath}}_O$ direction was measured when the total energy of the football fell below 20 J. This gave a reasonable approximation of the distance that the football would travel by the time it would come to rest. Figure 7 shows the results grouped into three distance ranges, while Fig. 8 shows a higher resolution plot.

In some cases, the limited range of the bounce model was insufficient to handle the motion of the football. Such cases were identified by nonphysical behavior – the football traveling into the ground after a bounce or the total energy of the football increasing during a bounce. Such cases were excluded from the analysis and are indicated in Figs. 7 and 8 as the white regions.

Figure 7 shows that distance traveled generally decreased as ϕ_0 and $(\omega_y)_0$ increased within the plotted range. For the higher values of ϕ_0 and $(\omega_y)_0$, there is also significant variation in distance traveled. For lower values, such as $\phi_0 < 55°$ and $(\omega_y)_0 < 10$ rad/s, there are larger regions of relatively consistent distance. This indicates that for some regions of initial conditions, the distance traveled is sensitive to the initial conditions, while for other regions, the distance traveled is relatively consistent.

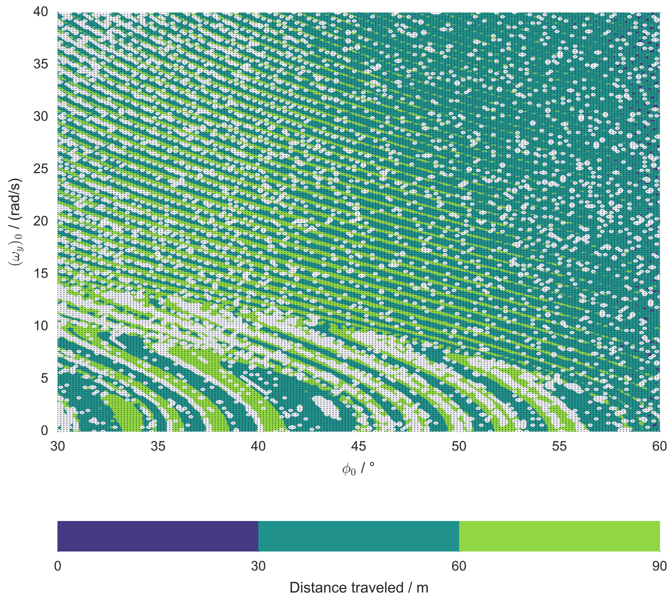

FIGURE 7: TOTAL DISTANCE TRAVELED AS A FUNCTION OF INITIAL CONDITIONS. INDIVIDUAL DATA POINTS ARE INDICATED WITH SMALL BLACK DOTS.

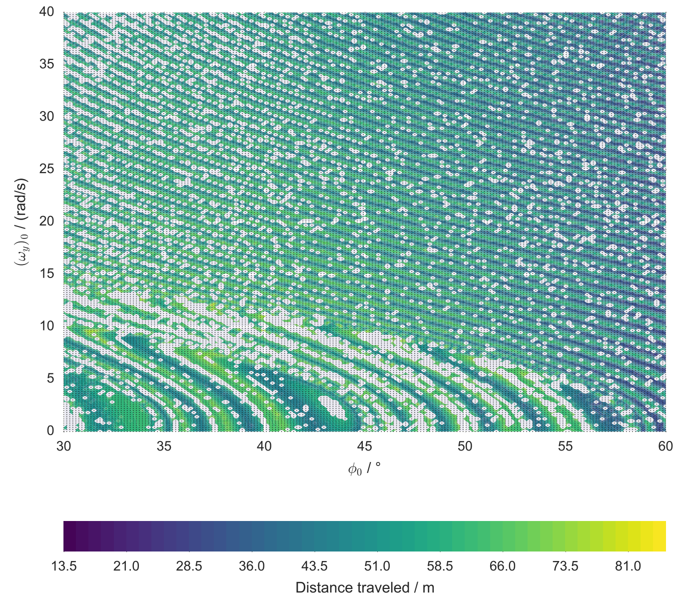

FIGURE 8: MORE DETAILED VERSION OF FIG. 7.

CONCLUSIONS

A model for the motion of an American football was developed, including flight and bouncing, and was simulated for a range of initial conditions. The results showed that in some areas of initial conditions, the distance traveled was relatively sensitive to the precise initial conditions, while in other areas, the distance traveled was more consistent even with small variations in initial conditions. This shows that a kicker could potentially target one of the larger regions, depending on the specific conditions of the game, to more reliably make a successful kick.

More research needs to be done to generalize the bounce model to three dimensions; in three dimensions, gyroscopic and tilt effects become important. Additionally, more data is needed to account for a wider range of velocities and angular velocities in order to apply this model to generate numerically accurate predictions. However, the results show qualitatively that this approach, with a quantitatively accurate bounce model, could be practically applied to develop a guide for punters to optimize their kicks. With such a guide and sufficient practice, punters could select and target the larger regions of initial conditions that produced desirable behavior, which would improve their chances of successful punts.

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