Sensitivity of Final Field Position to the Punt Initial Conditions in American Football

James D. Turner and Brian P. Mann



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1 Introduction

2 Math Model

- Flight
- Collision Detection
- Bouncing

3 Numerical Studies



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Introduction





Figure: Photograph of a football punter, Zoltán Meskó¹

 $^{^1 \ \}mbox{``Fourth play''}$ by cgilmour on Flickr is licensed under CC BY 2.0.



- Important characteristics of football kicks
 - distance traveled before impact
 - hang time in the air
 - distance traveled after bouncing
- Imprecise control over initial conditions
- Flight and bouncing of a football are highly nonlinear
- Are there large regions of initial conditions that have final distances relatively insensitive to initial conditions?





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Coordinate Systems



- Body-fixed coordinate system $B = \left\{ \hat{i}_B, \hat{j}_B, \hat{k}_B \right\}$
- Global coordinate system $O = \left\{ \hat{i}_O, \hat{j}_O, \hat{k}_O \right\}$
- Orientation of B relative to O represented as a quaternion







State of the football represented by

$$\boldsymbol{x} = \begin{bmatrix} x_O \ y_O \ z_O \ v_{xB} \ v_{yB} \ v_{zB} \ \omega_x \ \omega_y \ \omega_z \ e_0 \ e_1 \ e_2 \ e_3 \end{bmatrix}^\top$$

Motion of the ball was described by a first order ODE

$$\dot{x} = f(x)$$

using an empirical model from Lee, et al. (2013).

- Model accounted for:
 - gravity
 - aerodynamic forces (drag, lift, and yaw)
 - aerodynamic pitching moment (roll and yaw moments assumed to be negligible)



Collision when





- Highly nonlinear because of ellipsoidal ball
- Rough empirical model based on interpolation/extrapolation of data from Cross (2010)
- Limited to the x_O-z_O plane
- Model accounted for
 - orientation
 - velocity
 - angular velocity

but not variation in mechanical properties of football or turf



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Initial Conditions





Sample Trajectory





13 / 19

Sample Bounce





Sample Energies





Distance Traveled





Distance Traveled







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- Model for flight and bouncing
- Variation in sensitivity to initial conditions
- Kicker could target large desirable region
- Future: 3-D bounce model with wider range of velocities

